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In-Network Evaluation of Mobile Handset Performance

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Abstract

This paper investigates the use of the measurement reports transmitted on the Abis interface of the GSM network for evaluating handset performance. The most important advantage of this method, is the relatively easy access to a large number of calls made by many different users in the environments where handsets are expected to work.

A measurement campaign was carried out where controlled calls were made by 12 users in two different environments and with 4 different commercially available handsets. The calls were made to a normal GSM network where data logging equipment was connected to the Abis.

Large differences were found in the performance of the handsets. For example, the link loss was found to vary 14 dB from user to user for one handset, while for another it only varied about 5 dB under the same conditions.

1. Introduction

An important issue in the design of a mobile handset is the antenna, or the radiating elements. A poorly performing antenna can lead to increased power consumption and reduced coverage, and the network as a whole will lose capacity if many handsets have poor instead of high performance antennas.

The antenna performance is not included in the test for GSM type approval, but it is known that different handset antennas can perform widely different in real use [1], and thus the overall performance vary significantly among handset models.

Because of the complicated propagation environment of the antenna, well known antenna design parameters such as the radiation pattern of the antenna in free space conditions turn out to be insufficient in the search for good antenna designs. It is now widely recognized that the handset antenna performance needs to be evaluated in realistic scenarios,

where all important issues are included [6].

One method is to use propagation measurements such as those presented in [2] where measurements involving 200 test users are used to evaluate different antennas on a GSM-1800 handset. From these measurements it was concluded that the received power is highly user dependent, varying more than 10 dB from one user to another, and also that a patch antenna on the average is about 1 dB better than a dipole and about 3 dB better than a helix. Because of the differences observed from one user to another it is necessary to perform measurements with many different users in order to obtain a reliable average result.

Although performance evaluation using propagation measurements is possible, it is a common experience that these measurements are difficult to carry out in practice, and furthermore requires a lot of equipment. As an alternative it is possible to use the GSM system for the performance evaluation. As part of the normal operation of the network a number of parameters are measured regularly during calls by the base stations as well as by the handset. For example, the received signal strength is measured. In the network this information is transferred from the base stations to other parts of the network over the so-called Abis interface. A way to characterize the overall performance of a handset in the operating network is to collect the information transferred on the Abis during calls and process the data to obtain, for example, the average receive power level. In this way the actual GSM network becomes the measurement system and the performance evaluation can be done when the handset is in normal use. By logging data from the Abis interface it is possible to obtain data corresponding to a very large number of calls made by different users in different environments, and hence a statistical approach can be used to characterize the different types of handsets in the network.

In the current paper the two abovementioned methods for performance evaluation are compared. Before using the method of logging data from the Abis in order to character-

ize the performance of various handsets, it should be verified that the method yields results similar to those obtained in previous investigations [4, 3, 2]. To that end a new measurement campaign is carried out which is similar to the campaign in [2], but instead of propagation measurements, calls are made to a GSM network. Using special equipment, the control frames exchanged on the Abis interface are logged for each call in the campaign. By further processing of the logged frames, link quality information can be obtained.

The so-called link loss is obtained for different users making calls with 4 different handset types on 2 levels of an office building, where calls with both left and right hand are done. This allows a comparison of the handsets with respect to the average link loss and the variation of the link loss among the users, as well as analysis of the differences in performance obtained using left and right hand, and differences due to the floor of the building. In addition the body loss is also computed for all the handsets.

2. Abis Measurements

A GSM network is divided into a number of sub-systems each with a well defined function and interface to other sub-systems. Two overall types of information needs to be exchanged between the sub-systems, user traffic data (e.g. speech) and control information. The *Abis* interface is the interface between the base transceiver station (BTS) and the base station controller (BSC). Most of the information transferred between the network and the mobile station passes the *Abis*, in particular the measurement reports (MRs). The MRs contain, among other information, the RxLev and RxQual measurements, which is a measure of the received power level and the bit error rate, respectively. The MRs are transferred regularly and forms the basis for handover decisions.

The MRs are exchanged at least once per second, and usually about twice per second. The rate depends on whether other frames need to be transmitted on the link, since the capacity of the logical channel is limited and the MRs have lower priority than other types of frames. The MRs contains measurements for both the up-link and down-link where GSM handsets are required to perform the power measurements with an accuracy better than ± 4 dB. The RxLev parameter can take on 64 values and represents the received power in steps of 1 dB, where 0 represents a received power less than or equal to -110 dBm, and 62 represents a power between -49 dBm and -48 dBm. An RxLev value of 63 represents a received power above -48 dBm.

Although the MRs are intended for deciding about handovers, it is also a convenient tool for evaluating handset performance in realistic conditions. Since power control may be used it is important to know the transmitted power

when evaluating the received power. The MR frame transmitted on the *Abis* contains fields for the BTS transmit power level as well as an optional field for the MS transmit power level. The BTS transmit power level field indicates in steps of 2 dB from P_n dBm down to $P_n - 30$ dBm the power level used during the last measurement period, where P_n is the cell-specific maximum transmit power. Knowing the transmitted power, P_t and received power, P_r , the *link loss* can be defined as $L_l = P_t - P_r$, which is a quantity in dB. The present work uses L_l for comparing the performance of different handsets. Note that since P_n is unknown a value of zero is assumed. This means that the values of L_l presented in this paper may be offset by a constant from the correct values, but comparisons are still valid because P_n is the same for all the measurements.

2.1. Logging & Tracing Calls

Physically, the *Abis* interface is a pair of 2.048 Mbit/s links on which both the user traffic data and the control information is transferred. Each transceiver (TRX) in the BTS uses, in each up-link and down-link direction, one time-slot for control information and two time-slots for the user traffic data.

The *Abis* logging system consists of a PC with two special interface cards, each allowing the reception of the raw bits transmitted on a 2.048 Mbit/s link. Using some special logging software the PC extracts the GSM layer-3 (L3) frames from the received raw bits and stores all the frames on a large hard-disk for later processing.

Since each of the *Abis* time-slots carries control frames coming from and going to up to 8 air interface time-slots, the frames stored by the logging software is a mixture of frames belonging to different ongoing calls. Therefore some *call tracing* software have been developed which as input have the logged L3 frames and as output have an ordered sequence of frames belonging to each call. This software also takes care of following channel changes, handovers, etc.

All GSM handsets have a so-called international mobile equipment identity (IMEI) which is a 15 digit number unique for each handset. Because the IMEI is exchanged on the *Abis* in the beginning of each call it is possible to identify all calls made with a specific handset in the logged cells. When the relevant calls have been identified the tracing software can extract all information about the calls, including all the MRs.

2.2. Measurement Campaign

The measurements were carried out in a building at Aalborg University where a small base station is located approximately 75 m away. The base station carries both GSM-

900 and GSM-1800 cells, and data from both frequency bands are used. The measurement building is situated in the outskirts of the city and is a new four story office building mainly made of reinforced concrete with an outer brick-wall. Most inner walls are made of plaster board.

The measurements took place in corridors of the basement and the 1st floor. On the 1st floor the corridor provides access to small offices on both sides where the west-side offices have windows towards the base station. Approximately midway in the corridor to the east side there is an open laboratory space with windows away from the base station. Outside these windows there is open space with some large buildings approximately 75 m away. The 1st floor corridor is about 1.75 m–2 m wide, 29 m long, and 2.65 m high.

The corridor in the basement is about 1.75 m wide, 45 m long and 2.6 m–3.3 m high, and has storage rooms on both sides with no windows. About midway the corridor opens up and becomes a large open room with a small window towards east. In the south end the corridor ends in a stairwell where presumably most of the power enters, since this has windows and connection to other corridors with windows. The north end of the corridor has connection another corridor with an outdoor towards west in the other end.

Each measurement begins at the south end of the corridor and is made in the following way. The person enters a telephone number (see below) and waits for connection. When the user hears a voice in the handset, the user starts to walk slowly down the middle of the corridor while holding the handset in the way he or she feels is normal (Figure 1). When the corridor end is reached, the user turns around and returns to the starting point, where he or she ends the call. The time for the beginning and end of each call is noted, in order to allow identification of the calls in the logged data. Four different handsets are used and each handset is used with both left and right hand, so that each user makes 16 calls in total for the two floors.

The handsets are commercially available and represents some of the main handset types used today. In the following these are labelled A, B, C, D, where handsets have the following characteristics:

Handset A: A large handset with an external antenna.

Handset B: A small handset with an internal antenna.

Handset C: A large handset with an internal antenna.

Handset D: A small handset with an external antenna.

where the 'small' handsets are among the smallest handsets available today, about 10 cm by 4.5 cm, and the 'large' handsets are about 12 cm by 5 cm.

To avoid activation of discontinuous transmission (DTX) it is necessary to make sure that some speech/sound has to

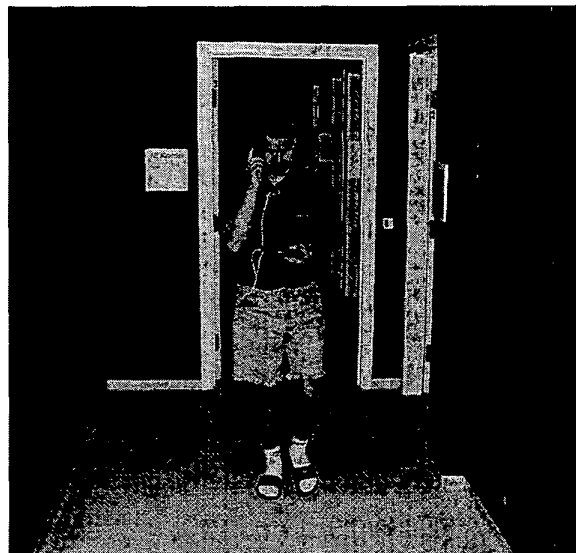


Figure 1. A user on his way down the basement corridor.

be transmitted. Regarding the downlink direction the measurements were made by calling a news service with a tape recorded voice. During the measurements the users carry a portable CD player connected to a small loudspeaker which is attached to the user close to the handset. The music from the loudspeaker deactivates the uplink DTX.

In total 25 test persons were used for these measurements, but due to a configuration error of the logging equipment, only data for the 12 first users are available for analysis, and furthermore for the 12th user only 1st floor measurements were obtained.

In addition to the user measurements, a number of free space measurements were performed where the handsets were fixed, using tape, to a wooden stick at an angle of approximately 54° from vertical. For each floor and handset combination the stick and handset was carried down the corridors, just as for the user measurements, except care was taken to hold the stick away from the body.

While all the controlled calls were made, Abis logging was carried out using the equipment described in Section 2.1. The Abis logging was made in cooperation with TeleDanmark, a Danish GSM network operator.

Note that all results presented below are for the downlink.

3. Results

3.1. Confidence Interval Estimation

In this work the mean link loss μ_{ll} is used as a measure to evaluate handset performance, where the mean is estimated based on the RxLev measurements and associated transmit power values obtained during a call as

$$\hat{\mu}_{ll} = \frac{1}{N} \sum_{i=0}^{N-1} L_l(i) \quad (1)$$

where $L_l(i)$ is the i th link loss measurement. It is known that the accuracy of the mean estimation $\hat{\mu}_{ll}$ depends on the number N of RxLev measurements and therefore the length of the call. $L_l(i)$ is a random variable and if N is large (say, greater than 20) then $\hat{\mu}_{ll}$ is approximately Gaussian distributed, according to the central limit theorem. The $100(1 - \alpha) \%$ two-sided confidence interval for $\hat{\mu}_{ll}$ is then $[\hat{\mu}_{ll} - S; \hat{\mu}_{ll} + S]$ where [5]

$$S = \frac{\hat{\sigma}_{ll} \cdot t_{\alpha/2, N-1}}{\sqrt{N}} \quad (2)$$

and $t_{\alpha, N}$ is the critical value of the t-distribution with N degrees of freedom at the critical level α . Thus in order to access the necessary number of measurements the standard deviation (SD) estimate $\hat{\sigma}_{ll}$ of the link loss has to be known. The best way to get this information is to estimate it from measured data.

For this purpose data was logged from an Abis interface of a GSM-900 BTS located near a shopping mall in a suburban area near Copenhagen. The data from 3 TRX units was logged for about 9 days, and the data contains traces of more than 100,000 connections (this includes actual telephone calls, location updates, dropped calls, etc.), and more than 200 different handset types as given by the first 6 IMEI digits.

From this data the link loss SD was estimated as follows. The traced calls involves handsets in many different locations and distances from the BTS, hence the overall link loss may be very different from call to call. Therefore, for each call trace the sequence of link losses is computed as well as the mean value, which is then subtracted so that all calls have a mean link loss of zero. It is then assumed that the SD is constant from call to call, so that the data from all calls made with the same type of handset can be combined when estimating the SD. This is of course a simplification of the real case, but suffices for a rough estimate.

The output of the above procedure is an estimate of the SD for each handset type observed. In practice only handset types represented with 300 or more calls are used, where some handsets are used much more often than others. The number of samples used in the estimation of the

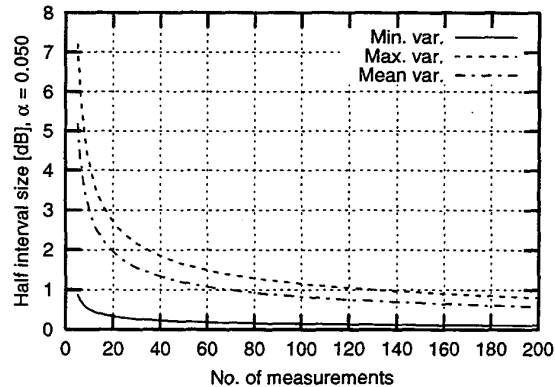


Figure 2. Confidence interval size in dB for mean link loss estimation.

SDs therefore varies from a few thousand samples to more than 100,000 samples.

The SD was estimated for 50 handset types. Figure 2 shows the S parameter defined in (2) for the minimum, the maximum, and the mean values of the SD estimated. The curves show that even for the maximum SD the confidence interval size is less than ± 1 dB if we have more than about 130 samples of the link loss. This corresponds to about 65 seconds of call time.

3.2. Repetition Measurements

Even though the measurements with the users are made in the same corridor and in the same way, there are inevitably small differences. In order to assess the magnitude of the resulting inaccuracies in the measurements, a series of repeated measurements were made with the same user in the basement corridor.

Measurements were made where the user walks as close as possible to the wall in the west side of the corridor and similarly measurements where the users walks close to the east side of the corridor. These measurements were made with the handset in the left hand. In addition a series of measurements were made where the user walks in the middle of the corridor, where both left and right hand is used. All these measurements were repeated 5 times with each of the four handsets.

Table 1 summarizes the results obtained. For each measurement series the table shows the mean value of the measured link losses,¹ as well as the SD. Some of the performed

¹In the following 'link loss' refers to the mean value of the series of link loss measurements obtained during one call.

Handset	East, Left			Middle, Left			Middle, Right			West, Left		
	Mean	Std	No.	Mean	Std	No.	Mean	Std	No.	Mean	Std	No.
A	88.3	0.2	5	84.9	1.5	5	87.6	0.5	5	85.3	0.3	5
B	92.8	-	1	91.7	1.0	4	89.6	0.9	5	91.6	0.6	4
C	93.1	1.0	5	93.7	1.0	5	88.1	1.0	5	93.9	1.1	5
D	86.8	0.9	5	86.1	0.7	5	87.1	2.6	2	87.2	0.5	5

Table 1. Statistics for repeated link loss measurements. All values are in dB, except the no. of measurements.

calls are not included in the results because a handover to another cell, where the Abis was not logged, was performed during the call (see also Section 3.4). In the table the 'No.' column shows the number of successful calls that were actually included in the calculations.

The mean values may be used to assess the sensitivity of the measurements to whether the users deviate from the path precisely in the middle of the corridor. Looking only at the left hand measurements, the maximum difference in the east, west, and middle mean values are for each of the handsets:

A: 3.4 dB, B: 1.2 dB, C: 0.8 dB, D: 1.1 dB

Thus, except for handset A, it seems that the exact path the user follows in the corridor is of less importance. The results for handset A seem strange and are likely due to a measurement error, *e.g.* the right hand was used instead of the correct left. The results presented below for the free space case also indicate that the measurements with handset A should have the same accuracy as the other handsets.

Regarding the variation among the repeated measurements with the same handset and position in the corridor, the SD is less than or equal to 1.5 dB in all but one case. The exception is handset D, middle/right which has a SD of 2.6, but in this case only two of the measurements were completed so the result is not as reliable as the others.

3.3. Free Space Measurements

The free space measurements were made on all the floors of the building and each measurement was repeated 5 times. Table 2 shows the mean and SDs obtained for these measurements.

The variation among the repeated measurements is low, as indicated by the SD which has a maximum value of 0.8 dB for any handset/floor combination. It is noticed that the SDs seem generally to be lower than for the repeated measurements including a user. This may be because the presence of a user introduces more sources of variation.

From the table it is also noted that for all floors handset B has the largest link loss, 2.6 dB larger than the second largest in the basement and more than 4 dB larger on

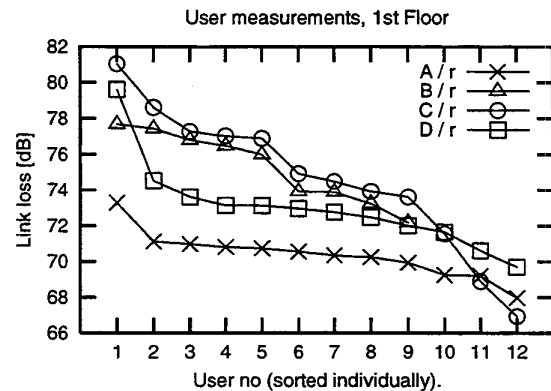


Figure 3. The link loss measured with different users for the four handsets on the 1st floor. Right hand is used in all cases.

all other floors. The other handsets have more equal link losses, although there are differences.

As expected, the link loss is generally higher, about 10 dB, in the basement compared to the values observed on the other floors.

3.4. User Measurements

Figure 3 shows the link losses measured on the 1st floor for the different users of the four handsets. Note that the results have been sorted for decreasing link loss individually for each handset type. It is obvious that the link loss varies significantly among the users, as shown in Table 3 where the maximum difference in link losses are given for both levels and all handsets. The figures in the table are in good agreement with the results found in [2], where user to user variations up to 10 dB were found for a handset with an external helix antenna, 6–7 dB differences were found for a patch and a whip antenna. Similar results were reported

Handset	Basement			1st Floor			2nd Floor			3rd Floor		
	Mean	Std	No.	Mean	Std	No.	Mean	Std	No.	Mean	Std	No.
A	78.8	0.6	5	68.5	0.2	5	66.2	0.2	5	68.3	0.4	5
B	82.4	0.8	4	73.0	0.3	2	70.5	0.1	3	72.3	0.0	2
C	79.7	0.4	2	67.1	-	1	64.7	0.3	3	66.7	0.1	2
D	78.5	0.2	3	68.7	0.7	2	65.6	0.2	2	68.0	0.3	2
Mean	79.9	-	-	69.3	-	-	66.8	-	-	68.8	-	-

Table 2. Statistics for free space link loss measurements. All values are in dB, except the no. of measurements.

Handset	Basement		1st Floor	
	Left	Right	Left	Right
A	5.9	6.2	3.0	5.3
B	5.5	3.2	5.2	5.5
C	4.7	4.5	10.4	14.1
D	3.7	4.3	3.7	9.9

Table 3. Maximum difference in link loss values for user measurements. All values are in dB. See also Table 4.

in [3].

It may be misleading only to look at peak differences, so Table 4 shows the SDs as well as mean values for the user measurements. The SDs are comparable to the ones reported in [2], although handset C has exceptionally large values on the 1st floor. A general view can be obtained by computing the mean of the SD over the floors, in which case the handsets with internal antennas have a larger SD than the handsets with external antenna. One might expect this since the internal antennas are closer to the user's hand, at least for the small handset B.

The mean link losses are also plotted in Figure 4 where the SDs are given as error bars. As expected the link loss is about 14 dB larger for the basement than for the 1st floor.

In most cases the handsets with internal antennas have a larger link loss than those with external antennas. Furthermore, the small handset with internal antenna in all cases has the largest mean link loss.

The results also show that handsets with external antennas are somewhat more sensitive towards which hand is used than handsets with internal antennas, most clearly seen in the basement.

The body loss is defined as the difference in received power when a user is present as compared to the free space case. In Figure 5 the mean body loss is plotted, where also the SD is indicated as error bars. The SD is the same as for the link loss case in Table 4.

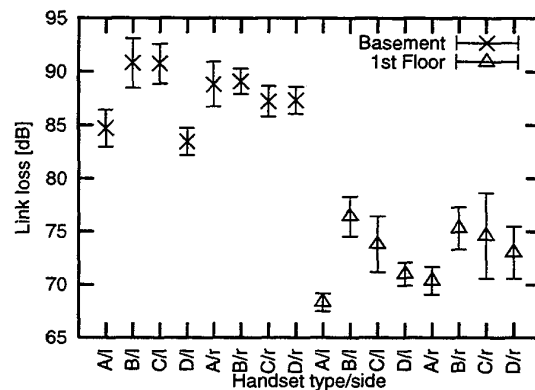


Figure 4. Mean link loss for all combinations of handsets, sides, and floors. The SDs are shown as error bars.

For handset A and D with external antennas the mean body losses are always smaller for left hand use than for right hand use. This may be because the antennas are located to the left on the top of the handset, and therefore the antenna is closer to the head and ear when held in the right hand than in the left hand. Also, from pictures it seems that the user's index finger is more often close to the antenna when held in the right hand. This corresponds well with the results of [4] where a close correlation was found between transmit power and how much of the top of the handset the user is covering.

It is interesting to see that for the right hand in the basement, handset B has the largest mean link loss, but actually has the smallest mean body loss. So, the large link loss must be due to the handset itself. Also note that although the mean body loss is positive in all cases, as one would expect, some of individual user measurements actually have a negative body loss for handset A on the 1st floor. This is

Handset	Basement, Left			Basement, Right			1st Floor, Left			1st Floor, Right		
	Mean	Std	No.	Mean	Std	No.	Mean	Std	No.	Mean	Std	No.
A	84.7	1.7	10	88.8	2.1	11	68.3	0.8	12	70.4	1.3	12
B	90.8	2.3	4	89.1	1.2	9	76.4	1.9	12	75.3	2.0	9
C	90.7	1.9	6	87.2	1.4	11	73.8	2.6	12	74.6	4.0	12
D	83.4	1.3	11	87.3	1.3	10	71.0	1.1	11	73.0	2.4	12

Table 4. Link loss statistics for user measurements. All values are in dB, except the no. of measurements.

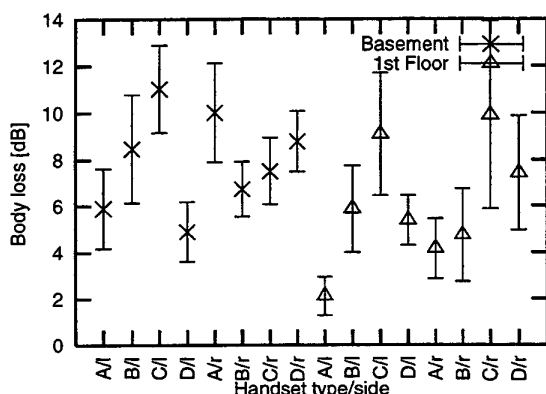


Figure 5. The mean measured body loss for all combinations of handsets, sides, and levels. The SDs are shown as error bars.

most likely due to measurement uncertainties which in combination with a low body loss results in a negative value.

Similar to previously, the number of measurements included in the averages are shown in the table. Only 11 users were measured in the basement, but the other missing calls are due to handovers during the call. It is striking that most calls are missing for handset B and C, which also are the ones with the largest mean link loss. So, one explanation for these outgoing handovers could be that the received power is too low for the logged BTS, but this has not yet been investigated fully.

4. Conclusion

The goal of the current work has been twofold. First of all to investigate whether the measurement reports on the Abis of the GSM network are useful for performance evaluation of handsets. And secondly, to evaluate 4 handsets representing some of the main types of handsets used to-

day, both a 'small' and a 'large' handset with an external antenna, and likewise two handsets with internal antennas.

A measurement campaign was conducted in which the four handsets were used by 12 test users on two floors of an office building. While a series of controlled user calls were made, all data transferred on the Abis interface was logged for later analysis. Using this data a number of parameters were extracted and compared with previously published results in order to find any resemblance. The results presented in this paper indeed seem to be very similar to the findings by other methods. Furthermore, the standard deviation of repeated measurements is better than about 1.5 dB.

As a measure of handset performance the so-called link loss is used, which is the mean value of the difference in transmitted and received power observed during each call. Large differences in the link loss was measured with the different users, where up to 14 dB was found for one handset on the 1st floor, but most in the range 3–6 dB.

Generally, the measured data seems to indicate that the variation in link loss due to the users is slightly larger for handsets with internal antennas, as compared to handsets with external antennas. Furthermore, the handsets with external antennas seem to have a smaller mean link loss than those with internal antennas, about 3–8 dB better when used in the left hand, but the difference is smaller (or even reversed) for right side use. Therefore, handsets with external antennas are more sensitive towards which hand is used. On the basement floor, the mean bodyloss increases about 4 dB from left to right hand usage for both handsets with external antenna, whereas for the handsets with internal antennas the change is 1–2 dB. On the 1st floor the changes are smaller, about 2 dB for the external antennas and 0.5–1 dB for the internal antennas.

The mean body loss ranges from about 2 dB up to 11 dB, where the external antennas result in lower body losses than internal antennas when used in left hand; in the right hand the situation to some degree reverses.

It should be noted that although the presented results generally show that handsets with external antennas perform better than handsets with internal antennas, this is probably very much dependent on the handset designs.

Hence, the results in [2] show that for another handset the external antennas (helix and dipole) have worse performance than an internal antenna.

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